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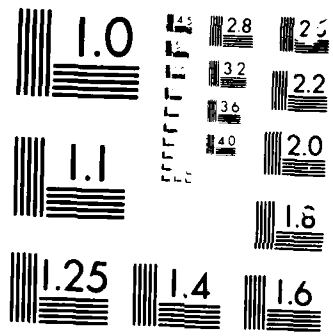
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FOREIGN TECHNOLOGY DIVISION

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THE EFFECT OF COUPLING IN AN INERTIAL NAVIGATION PLATFORM STABILIZED SYSTEM MADE UP OF THREE SINGLE-DEGREE-OF-FREEDOM GYROSCOPES

by

Di Bo-hao

FOREIGN TECHNOLOGY DIVISION

HUMAN TRANSLATION

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THE EFFECT OF COUPLING IN AN INERTIAL NAVIGATION PLATFORM STABILIZED SYSTEM MADE UP OF THREE SINGLE-DEGREE-OF-FREEDOM GYROSCOPES

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I. INTRODUCTION

We shall refer to three interacting single-axis systems as a three-axis system. If the interaction between the single-axis systems is weak, the three-axis system can be studied in the single-axis approximation. This approximation could lead to great simplification in the analysis and solution of the problem. On the contrary, if the interaction between the systems is strong, the single-axis approximation is not applicable to the three-axis system. To arrive at the criteria for the applicability of the single-axis approximation, it is necessary to study the effect of coupling between the systems.

One type of coupling involves the polar angle and the azimuthal angle. It depends on the geometry of the frame between the platform axes. This type of coupling can be eliminated by a coordinate transformation or an orthogonal transformation.

In addition, however, there is an output axis coupling in the semianalytic navigator made up of three single-degree-of-freedom gyroscopes. When the platform rotates around the gyroscope output axis relative to an inertial frame, the angular momentum vector $\vec{H}$ will rotate around the output axis in the platform frame since the angular momentum vector $\vec{H}$ is stable in the inertial frame. (Although the platform is not disturbed around the input axis, the angle sensor at the gyroscope output axis outputs a signal. This signal is amplified and transmitted to the input axis motor for platform control, causing the platform to rotate around the input axis. This rotation is not a counter-rotation against any disturbance. Nor is it the rotation produced by a correcting signal for tracking the earth coordinate system. It is an unwanted rotation due to the effect of coupling.)

II. THE EFFECT OF OUTPUT AXIS COUPLING IN A PLATFORM MADE UP OF SINGLE-DEGREE-OF-FREEDOM GYROSCOPES

1. Equations of a single-axis stable system rotating around the output axis

Figure 1 shows schematically a system with stable X_p axis. X_p is the input axis. β_x is the output axis. The direction of β_x is determined by the vector $\vec{H}_x \times \vec{X}_p$. The meaning is clear if we note that as the platform rotates around the positive X_p axis, the gyroscope will rotate around the output axis β_x. (Of course, we could have defined the direction of β_x as that of $\vec{X}_p \times \vec{H}_x$.)

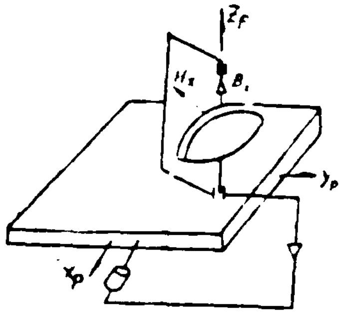

Figure 1

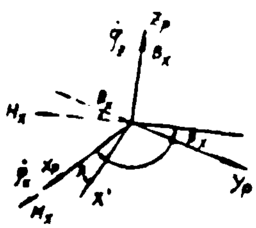

Figure 2

Thus we arrive at the gyroscope coordinate system $H_x X' \beta_x$ which is fixed in the body of the gyroscope. $X_p Y_p Z_p$ is the platform coordinate system. The relation between these two coordinate systems is shown in figure 2.

(1) <u>Equation of motion of platform</u>

If we consider only the stable axis, we can write down the following equation of motion:

[illegible] (1)

where J_x is the moment of inertia of the platform around the X_p axis, M_x is the perturbing torque acting on the stable axis, and M_D is the torque exerted by the X_p axis servo motor of the platform.

(2) Servo-motor equation

The torque exerted by the dc motor is

$$M_D = C_M \phi I_0 \tag{2}$$

where C_M is a parameter determined by the structure of the motor, ϕ is the motor magnetic flux, and I_0 is the electric current through the armature.

The voltage balance equation is

$$U = I_0 R_s + E \tag{3}$$

where U is the control voltage, R_s is armature resistance of the torque motor, and E is the back emf in the armature. In this equation

$$E = C_e \phi \Omega \tag{4}$$

where Ω is the angular velocity of the electric motor and C_e is a parameter determined by the structure of the motor.

(3) Amplifier equation

Let the transfer function from the output angle β_x of the fluid suspended integrating gyroscope to the mean output voltage of the servo amplifier be

$$\frac{U(S)}{\beta_x(S)} = K_x \cdot W(S) \tag{5}$$

where K_x is the product of all transfer functions.

(4) Fluid suspended integrating gyroscope equation

As can be seen from figure 2, we have the equation

$$H_x\dot{\varphi}_x \cdot \cos\beta_x = J_{\beta x}(\ddot{\varphi}_x + \ddot{\beta}_x) + C_x\dot{\beta}_x \tag{6}$$

where $J_{\beta x}$ is the moment of inertia of the gyroscope around the output axis and C_x is gyroscope damping coefficient.

Since β_x is very small, we can set $\cos\beta_x \approx 1$.

Since the rotation of the torque motor is in the direction that opposes the external perturbing torque, in equation (4) we have $\Omega = -\dot{\varphi}_x$. Under a Lagrange transformation, the above equations yield the following single-axis stable system equations

$$H_x \cdot S\varphi_x - J_{\beta x} \cdot S^2 \cdot \varphi_x =$$
$$S(J_{\beta x} \cdot S + C_x)\beta_x$$
$$J_x \cdot S^2\varphi_x + H_x \cdot S \cdot \beta_x = M_x - M_D$$
$$M_D = \frac{C_M\Phi}{R_D} \cdot U + \frac{C_e C_M\Phi^2}{R_s} S\varphi_x$$
$$U = K_x \cdot W_x(S) \cdot \beta_x$$

The corresponding block diagram is shown in figure 3.

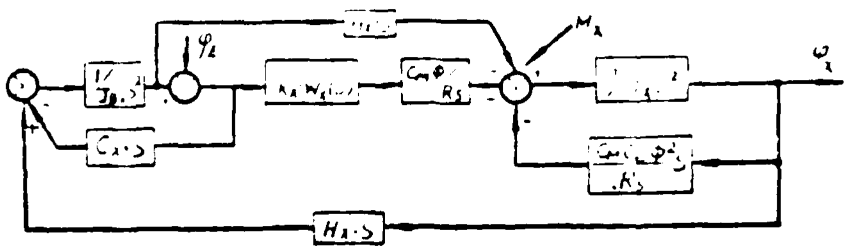

Figure 3

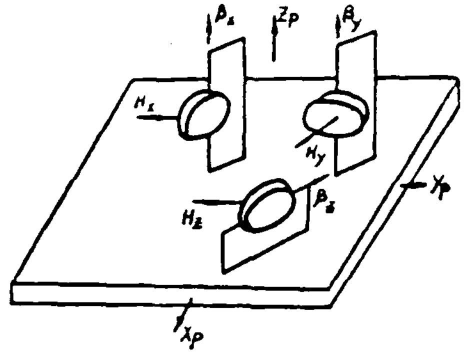

Figure 4

Let the three gyroscopes be positioned as in figure 4. The precession of the H_y axis will cause the component $H_y \sin \beta_y \dot{\beta}_y = H_x \dot{\beta}_y \beta_y$ of the gyroscope torque $H_y \dot{\beta}_y$ to act on the platform in the positive X_p direction (see figure 5). Similar effects associated with the other axes also exist. Obviously, since β_y is very small and $\dot{\beta}_y$ is also small, the effect of this type of coupling is very small and can be neglected.

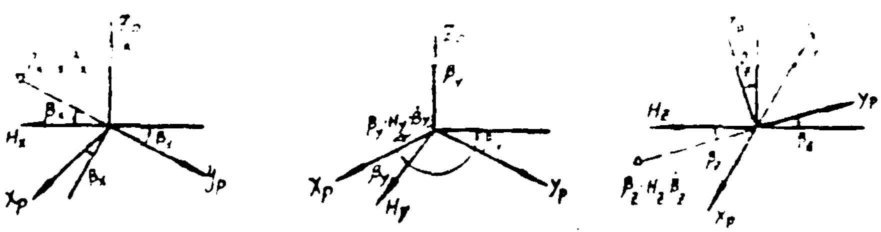

Figure 5

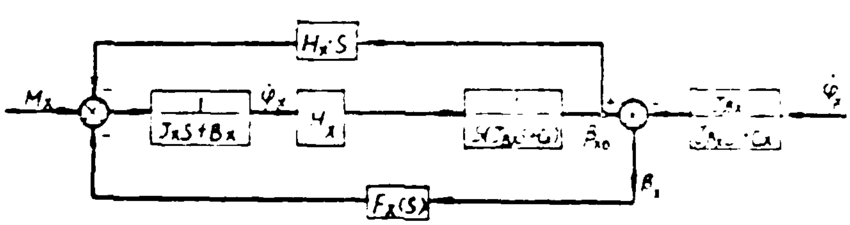

Figure 6

In figure 3, if we set $K_x W_x(S) C_M C_\phi / R_S = F(S)$ and $C_e C_M \phi^2 / R_S = \beta_x$, then figure 3 will be transformed to figure 6.

From figure 6 we have

$$\beta_x = \beta_{x0} - \frac{J_{ax}}{J_{ax} \cdot S + C_x} \cdot \dot{\varphi}_z \tag{7}$$

The sensor output signal β_x to the amplifier consists of two terms. The first term is precisely the signal which removes the effect of perturbation of the platform around the X_p axis. The second term is the extraneous signal introduced into the X servo loop by the platform rotating around the Z_p axis. The second term on the right-hand side of equation (7) represents the effect of this type of coupling.

2. Mathematical model of the three-axis system

Let the three gyroscopes be positioned as in figure 4. After obtaining the stable system equations for the X_p, Y_p, and Z_p axes, we combine the three loops and, as a result, obtain the block diagram of the coupled three-axis system as shown in figure 7.

It can be seen from figure 7 that the angular velocity $\dot{\varphi}_z$ of the platform rotating around the Z gyroscope input axis is coupled to the sensors of the X and Y gyroscopes. The rotation around the X gyroscope input axis is coupled to the sensor of the Z gyroscope. But the rotation around the Y gyroscope input axis is not fed back to the sensor of the Z gyroscope. Therefore, the Y servo loop is

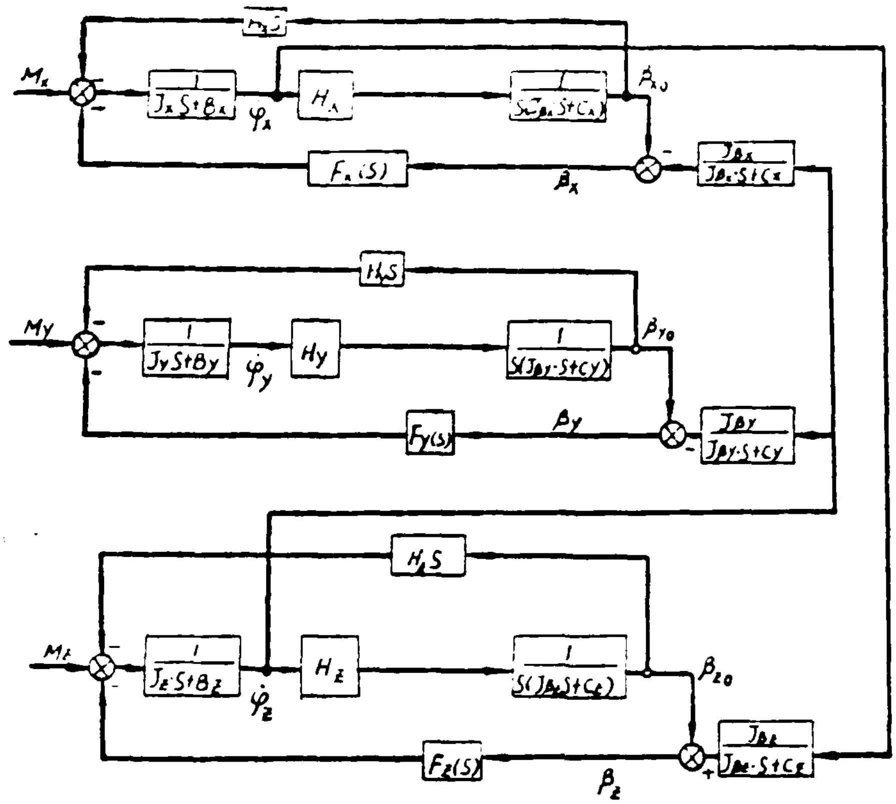

Figure 7

perturbed only by the rotation of the platform around the Z_p axis. This does not lead to any instability. The angular velocities $\dot{\varphi}_x$ and φ_z around the X_p and Y_p axes are coupled respectively to the angular displacements β_x and β_z around the gyroscope output axes, resulting in a closed loop. Apart from the mutual perturbation between the axes, there will also be some effect on system performance. However, if the coupling is so small that the effect on system performance can be negligible, then analysis of the system will be greatly simplified. In the following sections, we will study the conditions which minimize the effect of coupling.

3. Negative feedback condition for coupled closed loops

If there is positive feedback in the coupled closed loop, then the effect of coupling will increase with platform motion. To minimize the effect of coupling, we must first impose a negative feedback condition in the coupled closed loop. This condition leads to the following gyroscope positioning requirements:

(1) Gyroscope positioning as shown in figure 4

Platform rotation around the positive X axis will cause the Z gyroscope sensor to output an extraneous signal. Suppose the signal is along the positive β_z direction, so that the platform rotates around the negative Z_p axis. The sensor of the X gyroscope then outputs an extraneous signal along the positive β_x axis, causing the platform to rotate around the negative X_p axis.

(2) Gyroscope positioning as shown in figure 5

Platform rotation around the positive X_p axis will cause the Z gyroscope sensor to output an extraneous signal. The signal is along the negative β_x direction, so that the platform rotates around the positive Z_p axis. The sensor of the X gyroscope then outputs an extraneous signal, causing the platform to rotate around the positive X_p axis.

Summarizing the above discussion, we note that the effect of coupling exists in the servo loop made up of two parallel

principal axes which form a closed loop. The H directions of the two gyroscopes must be the same in order to reduce the effect of coupling.

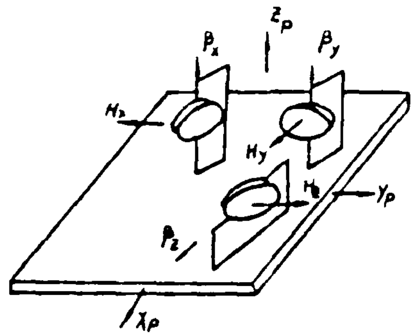

Figure 8

4. The condition for minimum output coupling

We will now derive the conditions which minimizes the effect of coupling to a negligible level.

Using the block diagram of figure 7, we now write

$$\beta_x = \frac{H_x \dot{\varphi}_x}{S(J_{Bx} \cdot S + C_x)} - \frac{J_{Bx}}{J_{Bx} \cdot S + C_x} \cdot \dot{\varphi}_z \tag{8}$$

If, within a meaningful range of frequencies, we have

$$\left| \frac{H_x \dot{\varphi}_x}{S(J_{Bx} \cdot S + C_x)} \right| \gg \left| \frac{J_{Bx}}{J_{Bx} \cdot S + C_x} \cdot \dot{\varphi}_z \right| \tag{9}$$

then the second term on the right-hand side of equation (8) can be neglected. The effect of coupling can then be neglected.

Equation (9) yields

$$\left|\frac{H_x \dot{\varphi}_x}{S}\right| \gg |J_{Bx} \cdot \dot{\varphi}_z| \tag{10}$$

In equation (10), $\dot{\varphi}_x$ consists of two terms:

$$\dot{\varphi}_x = \Delta\dot{\varphi}_x + \delta\dot{\varphi}_x$$

where $\Delta\dot{\varphi}_x$ is the angular velocity increment arising from the perturbation M_x and $\delta\dot{\varphi}_x$ is the angular velocity increment arising from coupling.

Similarly, we have

$$\dot{\varphi}_z = \Delta\dot{\varphi}_z + \delta\dot{\varphi}_z$$

Substituting in equation (10), we have

$$|H_x(\Delta\dot{\varphi}_x + \delta\dot{\varphi}_x)/S| \gg J_{Bx}|\Delta\dot{\varphi}_z + \delta\dot{\varphi}_z|$$

That is, we have

$$H_x \gg J_{Bx}|\Delta\dot{\varphi}_z + \delta\dot{\varphi}_z/\Delta\dot{\varphi}_x + \delta\dot{\varphi}_x|\omega_b$$

where ω_b is the cutoff frequency of the system.

If we let

$$A_x = |\Delta\dot{\varphi}_z + \delta\dot{\varphi}_z/\Delta\dot{\varphi}_x + \delta\dot{\varphi}_x|$$

then, we have

$$H_X/J_{gX} \gg A_X \cdot \omega_b \tag{11}$$

If the errors in the angular velocity and the coupling effect arising from perturbation along the axes are of comparable magnitude, then $A_X \approx 1$. Equation (11) becomes

$$H_X/J_{gX} \gg \omega_b$$

If the effects are not comparable, then, in general, rolling is greater than yawing, that is, $\Delta\dot{\varphi}_X > \Delta\dot{\varphi}_Z$. Consequently, equation (11) is even more easily satisfied.

Similarly, we have

$$H_Y \gg \left| \frac{\Delta\dot{\varphi}_Z + \delta\dot{\varphi}_Z}{\Delta\dot{\varphi}_Y + \delta\dot{\varphi}_Y} \right| \cdot J_{gY} \cdot \omega_b$$

$$\Rightarrow \frac{H_Y}{J_{gY}} \gg A_Y \cdot \omega_b \tag{12}$$

$$H_Z \gg \left| \frac{\Delta\dot{\varphi}_X + \delta\dot{\varphi}_X}{\Delta\dot{\varphi}_Z + \delta\dot{\varphi}_Z} \right| \cdot J_{gZ} \cdot \omega_b$$

$$\Rightarrow \frac{H_Z}{J_{gZ}} \gg A_Z \cdot \omega_b \tag{13}$$

Since rolling dominates, $\Delta\dot{\varphi}_X$ is greater than either $\Delta\dot{\varphi}_Y$ or $\Delta\dot{\varphi}_Z$. Therefore, in equation (13), $A_Z > 1$. The condition expressed in equation (10) is more severe than those involving the other two axes. This demands a greater H for the Z gyroscope than for the other two gyroscopes.

If the errors arising from pitching and yawing are comparable, we let Y_p be the roll axis and let X_p be the pitch axis. Then, in equations (11), (12), and (13), A_x, A_y, and A_z are all approximately equal to 1, thus relaxing these conditions. Let H be the angular momentum of each gyroscope, J_g be the moment of inertia around the output axis, and ω_b be the approximate cutoff frequency of the loop in a single-axis approximation. If $H/J_g \gg \omega_b$, the effect of coupling will be negligibly small. To realize the condition $H/J_g \gg \omega_b$, it is advantageous for two gyroscopes with parallel principal axes to be positioned so that their axes are along the roll axis.

From the condition $H/J_g \gg \omega_b$, it is obvious that the effect of coupling is reduced by a reduction in the moment of inertia J_g of the gyroscope around the output axis, by a reduction in the loop cutoff frequency in the single-axis approximation, and by an increase in H.

III. CONCLUSION

In summary,

1. The effect of coupling exists between the stable loops of two gyroscopes whose principal axes are parallel. To minimize the effect of coupling, it is advantageous to position the gyroscopes so that their principal axes H are parallel;
2. To minimize the effect of coupling, it is advantageous for two gyroscopes with parallel principal axes to be positioned so that their principal axes H are along the roll axes;

3. For each single-axis loop, the effect of coupling is reduced by a reduction in the moment of inertia J_g of the gyroscope around the output axis, by a reduction in the loop cutoff frequency ω_b in the single-axis approximation, and by an increase in the angular momentum H of the gyroscope. The effect of coupling is negligible if the condition $H/J_g \gg \omega_b$ is satisfied.

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